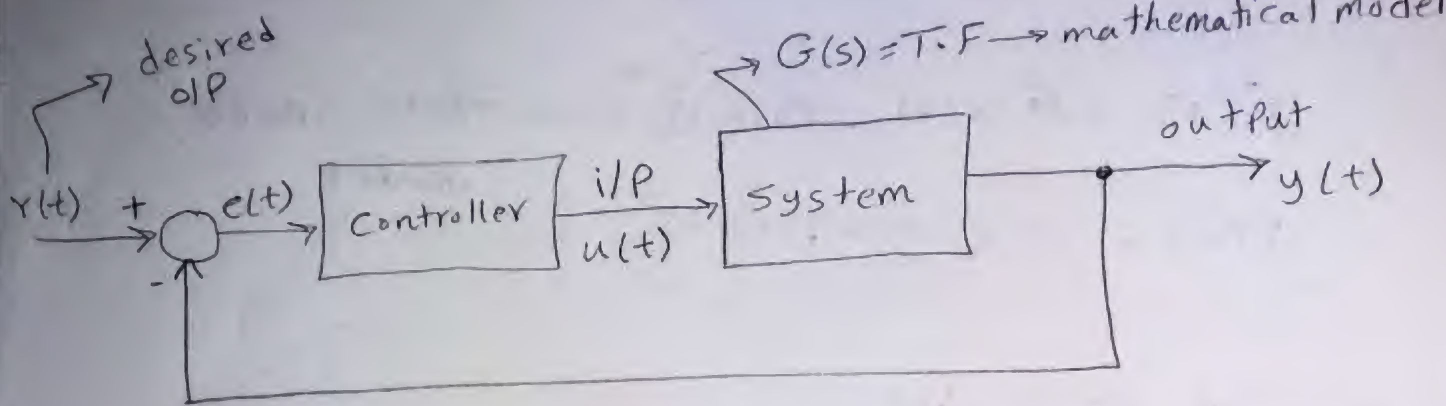
٥/٦ محاضرة نظم وإشارات

ع

→ state space representation (model)



controller

- → classical (conventional)
- → traditional
- → PID
- ⋮

Modern Controller

state-space model

→ mathematical model :-

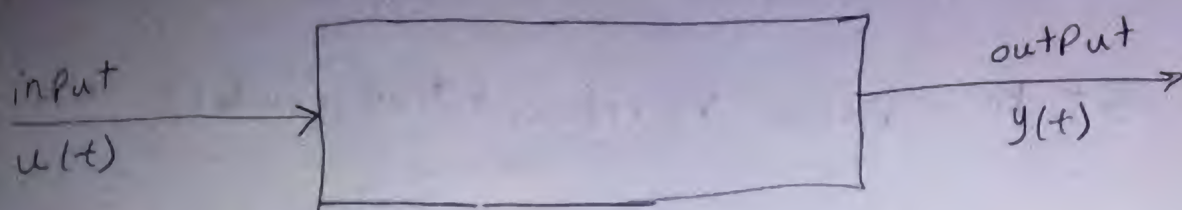
(1) T.F $\quad G(s) = \frac{Y(s)}{U(s)}$

(2) Block diagram.

(3) signal flow graph.

(4) Differential eqn. $\quad \boxed{5}$ state-space model.

system = Process = Plant

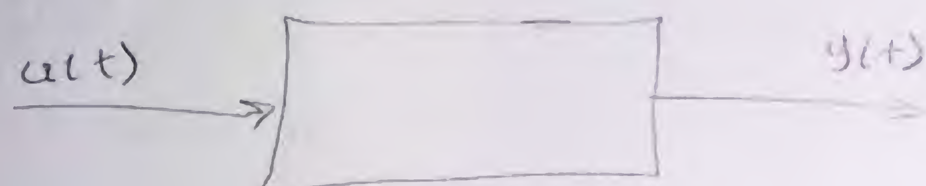


$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) + D\,u(t)$$

$\longrightarrow$ state-space model

$\Rightarrow$ system may be

1) SISO $\equiv$ single input single output



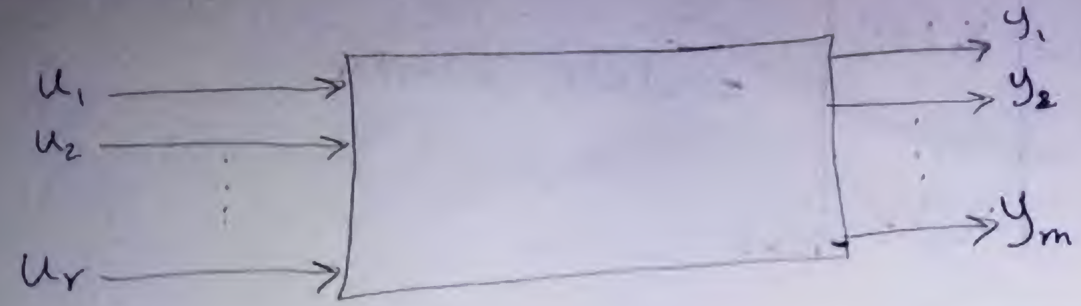
2) SIMO $\equiv$ single input multi output.

m = no of outputs

3) MISO $\equiv$ multi input single output

$r \rightarrow$ no. of inputs

4) MIMO



* if system is "SIMO" output will be:-

"مصفوفة بها عمود واحد"

column vector

$$y(t) = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \Rightarrow \text{m-dimensional output vector.}$$

* for "MISO"

$$\begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix} = u(t) \longrightarrow \text{input vector}$$

⇒ state variables :-

→ makes it easy to determine performance of behavior of system.

→ $x_1(t),\ x_2(t) \ldots\ldots,\ x_n(t)$

$n \to$ no. of state variables (states)

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}_{n*1} = \text{state vector}$$

$$\dot{X}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix}_{n*1}$$

→ we will deal with "SISO".

1) $A = [\quad]_{n*n}$ = state (system) matrix

2) $B = [\,|\,]_{n*1}$ ≡ input matrix.

3) $C = [\,\text{———}\,]_{1*n}$ ≡ output matrix.

4) D_{1*1} = scaler = direct transmission value

= feed forward value in vector-matrix form.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}_{n*1} = \underbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{n*n}}_{A} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}_{n*1} + \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}_{n*1}}_{B} * u(t)_{1*1}$$

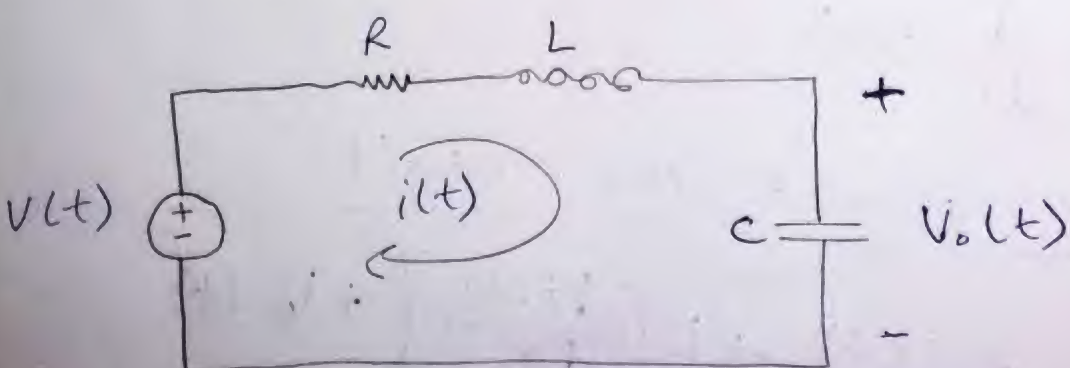
$$y(t)_{1*1} = \underbrace{\begin{bmatrix} & & \end{bmatrix}_{1*n}}_{C} \begin{bmatrix} x_1(t) \\ \vdots \\ x_2(t) \end{bmatrix} + D_{1\times 1} * u(t)$$

→ Dynamic eqns.

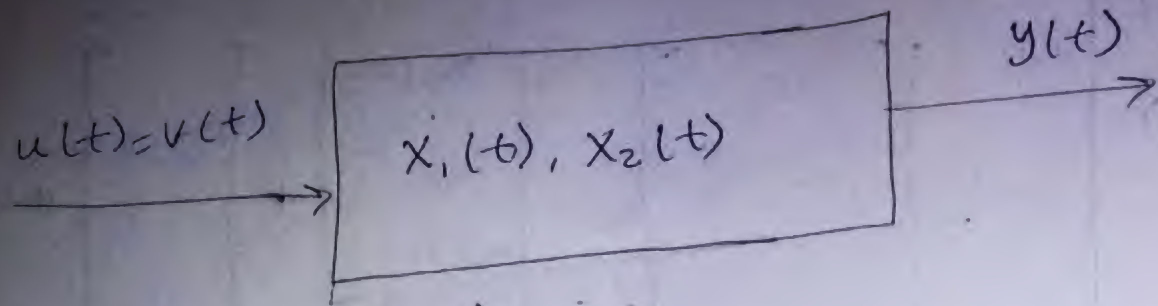
1) $\dot{x}(t) = A\,x(t) + B\,u(t)$ → state eqn.

2) $y(t) = C\,x(t) + D\,u(t)$ → output eqn.

→ Example (1) [Electrical system].



→ write state-space model.



$$\boxed{\begin{aligned} x_1(t) &= i(t) \\ x_2(t) &= V_c(t) \end{aligned}}$$

$n = $ no. of states $= 2$

$$\underbrace{\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}}_{n*1 \,=\, 2*1} = \underbrace{\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}_{n*n}}_{2*2} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}_{n*1}}_{2*1} + \underbrace{\begin{bmatrix} \cdot \\ \cdot \end{bmatrix}_{n*1}}_{2*1} u(t)$$

$$y(t) = \underbrace{\begin{bmatrix} \cdot & \cdot \end{bmatrix}_{1*n}}_{1*2} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + D\, u(t)$$

$$V_L(t) = L \frac{di(t)}{dt} \quad , \quad i_c(t) = C \frac{dV_c(t)}{dt}$$

$$\dot{x}_1(t) = \frac{d}{dt} i(t)$$

→ <u>KVL around the circuit</u>

$$v(t) = R\, i(t) + L\left[\frac{di(t)}{dt}\right] + V_c(t)$$

$$\frac{di(t)}{dt} = \frac{-R}{L} i(t) - \frac{1}{L} V_c(t) + \frac{1}{L} V(t)$$

$$\therefore \quad \dot{x}_1(t) = \frac{-R}{L} x_1(t) - \frac{1}{L} x_2(t) + \frac{1}{L} u(t) \longrightarrow (1)$$

$$\dot{x}_2(t) = \frac{d}{dt} V_c(t)$$

$$i(t) = C \frac{dV_c(t)}{dt} \Rightarrow \frac{dV_c(t)}{dt} = \frac{1}{C} i(t)$$

$$\boxed{\therefore \quad \dot{x}_2(t) = \frac{1}{C} x_1(t)} \longrightarrow (2)$$

$$y(t) = V_o(t) = V_c(t) = x_2$$

$$\boxed{y(t) = x_2(t)} \longrightarrow (3)$$

نعمل بيهم مصفوفة ←

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u(t)$$

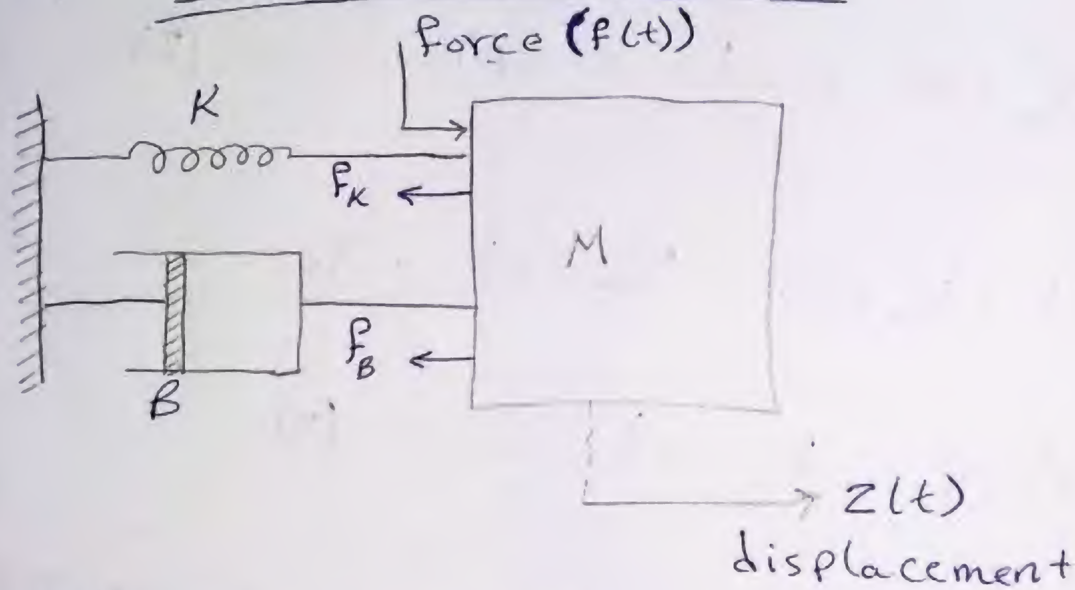
$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \underbrace{0}_{\to D\,u(t)}$$

ـــ شكل المسائل ممكن يكون ـــ ←

$$\dot{x}(t)_{2\times1} = \underbrace{\begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{B} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

<u>Example (2)</u> <u>Mechanical system</u>



<u>Find state space model</u>

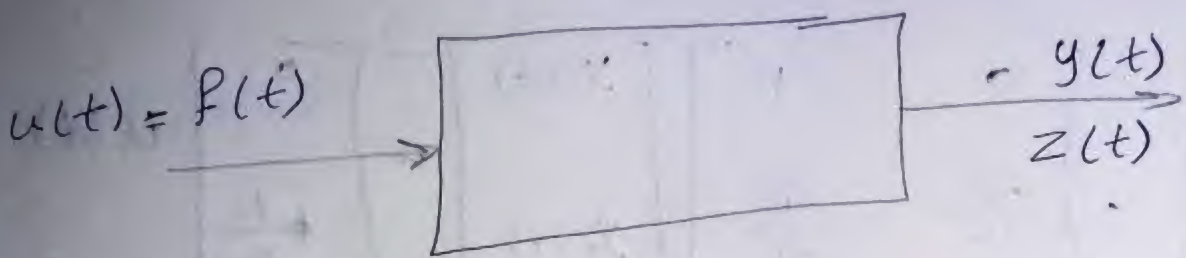
eqn of motion ⇒ differential eqn ⇒

Newton's second law.

$$M \cdot \ddot{z}(t) = f(t) - B\left[\dot{z}(t) - 0\right] - K\left[z(t) - 0\right]$$

$$M\ddot{z}(t) + \beta\dot{z}(t) + K z(t) = f(t)$$

n = no. of state variables $\Rightarrow \begin{cases} x_1(t) \\ x_2(t) \end{cases}$



$\ddot{z} \longrightarrow$ acceleration. $\dot{z} \longrightarrow$ velocity.

$$x_1(t) = z(t) \qquad , \quad x_2(t) = \dot{z}(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}_{2\times 2} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix}_{2\times 1} u(t)$$

$$y(t) = \begin{bmatrix} \quad & \quad \end{bmatrix}_{1\times 2} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + D\,u(t)$$

$$\dot{x}_1(t) = \dot{z}(t) \Rightarrow \boxed{\dot{x}_1(t) = x_2(t)} \longrightarrow (1)$$

$$\dot{x}_2(t) = \ddot{z}(t) = \frac{-K}{M} z(t) - \frac{\beta}{M}\dot{z}(t) + \frac{1}{M} f(t)$$

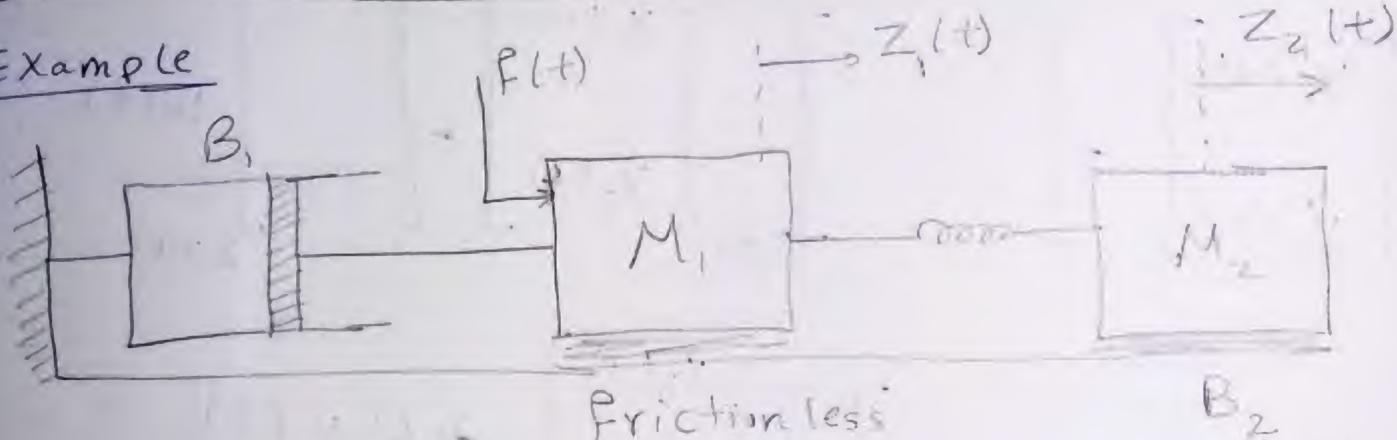
$$\boxed{\dot{x}_2(t) = \frac{-K}{M} x_1(t) - \frac{B}{M} x_2(t) + \frac{1}{M} u(t)} \rightarrow (2)$$

output $y(t) = z(t) = x_2(t)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-K}{M} & \frac{-B}{M} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \cancel{D}\, u(t) \rightarrow 0$$

Example



* For Mass M_1

$$\boxed{M_1 \ddot{Z}_1(t) + B_1 \dot{Z}_1(t) + K Z_1(t) = f(t) + K Z_2(t)}$$

* For Mass M_2

$$\boxed{M_2 \ddot{Z}_2(t) + B_2 \dot{Z}_2(t) + K Z_2(t) = K Z_1(t)}$$

$\boxed{n = 4}$

$$\overset{z_1}{\downarrow} \qquad\qquad \overset{z_2}{\downarrow}$$

$$z_1(t),\ \dot{z}_1(t) \qquad z_2(t),\ \dot{z}_2(t)$$

$$x_1(t) = z_1(t) \qquad x_3(t) = z_2(t)$$

$$x_2(t) = \dot{z}_1(t) \qquad x_4(t) = \dot{z}_2(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \underset{4*4}{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K}{M_1} & \frac{-B_1}{M_1} & \frac{K}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K}{M_2} & 0 & \frac{-K}{M_2} & \frac{-B_2}{M_2} \end{bmatrix}} \underset{4*1}{\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}} + \underset{4*1}{\begin{bmatrix} 0 \\ \frac{1}{M_1} \\ 0 \\ 0 \end{bmatrix}} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}_{1*4} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + 0\ u(t)$$

$$\boxed{\dot{x}_1(t) = x_2(t)}$$

$$\dot{x}_2(t) = \frac{-K}{M_1} x_1 - \frac{B_1}{M_1} x_2(t) + \frac{K}{M_1} x_3(t) + \frac{1}{M_1} u(t)$$

$$\dot{x}_3(t) = x_4(t)$$

$$\dot{X}_4(t) = \frac{-K}{M_2} X_3 - \frac{B_2}{M_2} X_4 + \frac{K}{M_2} X_1$$

output

$$y(t) = Z_2(t) = X_3(t)$$

ـــ تم تعديل القيم في الصفحة السابقة.